Bayes in Robotics

Here's a quick intro into how Bayesian techniques can be used in robotics, specifically with noisy sensors.

The Problem

You have a robot capable of moving among four rooms, identified as A,B,C, and D.

You have a room sensor capable of detecting which room you are in. The only problem is that it's not that accurate. Sometimes it will incorrectly tell you that you are off by one room, i.e. an adjacent room.

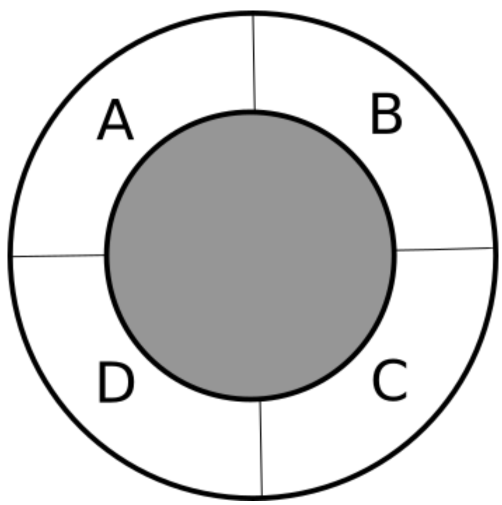

Figure 1: Circular Room Setup

Figure 1 shows the room layout. If, for example the robot is in room B then the sensor would indicate room B 80% of the time, Room A 10% of the time and Room C 10% of the time.

So, since we have no inital information on where we are lets just guess some probalilities on which room we are in.

If X is a arandom variable indicating the actual room the robot is in, our guesses are expressed as follows,

$$P(X = A) = 0.30$$
$$P(X = B) = 0.20$$
$$P(X = C) = 0.50$$
$$P(X = D) = 0.00$$

For example this is read as, "the probability that the robot is actually in room A is 0.30."

Or in a more convenient form in Table 1.

A	B	C	D
0.30	0.20	0.50	0.00

Table 1: Initial Room Probabilities

Now let Z be a random variable from the sensor indicating the room that our noisy sensor thinks the robot is in.

Now let's assume the robot is in room B.

We take a sensor reading.

The probabilities of our room location, based on our noisy sensor, are,

$$P(Z = A|X = B) = 0.10$$
$$P(Z = B|X = B) = 0.80$$
$$P(Z = C|X = B) = 0.10$$
$$P(Z = D|X = B) = 0.00$$

For example, this is read as, "the probability that the sensor says room B given that the robot is actually in room B is 0.80."

Again, in a more convenient form in Table 2.

Now that we have a room sensor reading we can apply Bayes Theorem to update our belief (from our prior random guess (look at Table 1 again) of where the robot is, expressed in a probability.

A	B	C	D
0.10	0.80	0.10	0.00

Table 2: Imperfect Sensor Data When Robot is in Room B

We would like to compute the new probability that the robot is in each room. That is,

$$P(X = A|Z = B)$$
$$P(X = B|Z = B)$$
$$P(X = C|Z = B)$$
$$P(X = D|Z = B)$$

Generally,

$$P(X|Z) = \frac{P(Z|X)\ P(X)}{P(Z)}$$

Let's get to it, first our updated probability that the robot is in room A after the sensor reading.

$$P(X = A|Z = B) = \frac{P(Z = B|X = A)\ P(X = A)}{P(Z = B)}$$

but,

$$P(Z = B|X = A) = 0.1 \quad \text{(See Table 2)}$$
$$P(X = A) = 0.3 \quad \text{(See Table 1)}$$

yielding, so far,

$$P(X = A|Z = B) = \frac{0.1 \cdot 0.3}{P(Z = B)}$$

Computing $P(Z = B)$ is a bit harder.

$$P(Z = B) = \sum_{i}^{N} P(Z = B|X = x_i)\ P(X = x_i$$

Or in our case,

$$P(Z = B) = (0.8 \cdot 0.3) + (0.1 \cdot 0.2) + (0.0 \cdot 0.5) + (0.1 \cdot 0.0)$$
$$P(Z = B) = 0.26$$

Resulting in,

$$P(X = A|Z = B) = \frac{0.1 \cdot 0.5}{0.26}$$

or,

$$P(X = A|Z = B) = 0.115$$

Now for the other rooms. Note we don't have to compute $P(Z = B)$ again.

$$P(X = B|Z = B) = \frac{P(Z = B|X = B)\ P(X = B)}{P(Z = B)}$$
$$P(X = B|Z = B) = \frac{0.8 \cdot 0.2}{0.26}$$
$$P(X = B|Z = B) = 0.615$$

$$P(X = C|Z = B) = \frac{P(Z = B|X = C)\ P(X = C)}{P(Z = B)}$$
$$P(X = C|Z = B) = \frac{0.1 \cdot 0.5}{0.26}$$
$$P(X = C|Z = B) = 0.038$$

$$P(X = D|Z = B) = \frac{P(Z = B|X = D)\ P(X = D)}{P(Z = B)}$$
$$P(X = D|Z = B) = \frac{0.0 \cdot 0.0}{0.26}$$
$$P(X = D|Z = B) = 0.0$$

The results are summarized in Table 3. Note we are somewhat confident we are in room B based on our sensor reading, even though we originally thought room C was out more likely location.

A	B	C	D
0.115	0.615	0.038	0.00

Table 3: Updated Belief of Our Room Location